turbulent diffusion as being responsible for the main part of the transport.

Finally it should be pointed out that the decrease of $v'/u^*$ at high Reynolds numbers found by Kjellström and Hedberg [5] is probably due to errors in the evaluation of the turbulent data. Later experience (see Kjellström and Hedberg [6]) indicates that the variations both of the exponent $c$ in Collis' law and the direction sensitivity coefficient $k^2$ as functions of velocity must be considered in the evaluation. This was not done in the earlier measurements of these authors referred to by Skinner *et al.* [1].

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# RELAMINARIZATION IN TUBES

D. M. McELIGOT,* C. W. COON† and H. C. PERKINS‡

Energy, Mass and Momentum Transfer Laboratory, Aerospace and Mechanical Engineering Department,
University of Arizona, Tucson

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## NOMENCLATURE

| | |
|---|---|
| $a, d$, | temperature dependence exponents for viscosity and specific heat, respectively; e.g. $\mu/\mu_i = (T/T_i)^a$; |
| $G$, | mass flow velocity, $4\dot{m}/\pi D^2$; |
| $H$, | enthalpy. |

Nondimensional parameters and variables

| | |
|---|---|
| $\overline{c_p}$, | specific heat, $c_{p,b}/c_{p,i}$; |
| $Q^+$, | local heat flux parameter, $q''_w D/(2k_b T_b)$; |
| $q^+$, | heat flux parameter, $q''_w/(G c_{p,i} T_i)$; |
| $Re$, | Reynolds number, $4\dot{m}/(\pi D \mu)$; |
| $\overline{T_b}$, | bulk static temperature, $T_b/T_i$; |
| $\overline{\mu}$, | viscosity, $\mu_b/\mu_i$. |

Subscripts

| | |
|---|---|
| $b$, | evaluated at bulk static temperature; |
| $i$, | initial, inlet; |
| 0, | stagnation conditions; |
| trans, | transition; |
| $w$, | wall, heat transfer surface; |
| $\infty$, | free stream. |

The absence of a subscript on gas properties indicates properties evaluated at gas bulk static temperature.

In recent years the transition from turbulent to laminar behavior has become a topic of interest in consideration of accelerated flows, as found in rocket nozzles, and of duct flows such as the heating channels of proposed nuclear rocket engines. For gaseous circular tube flow evidence of this effect appears in the early work of Humble *et al.* [1] and of Barnes [2]. An effect of such transition is shown in Fig. 1 which presents typical wall temperature data of Coon [3]. Predictions based on accepted constant and variable properties, turbulent flow correlations are shown for comparison. The transition causes a dangerous increase in wall temperature. In this note, recent work is discussed briefly, and the relationship between the transition for internal heat flows and for "external" accelerated flows is presented.

Independently, around 1963, Moretti and Kays [4], Launder [5] and McEligot [6] began to detail this transition process in a variable geometry duct, in a nozzle and in heated tubes, respectively. While it is likely that the process is a continuous one rather than a "critical" one, i.e. instantaneous change from one flow to the other, it is useful to

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* Professor of Aerospace and Mechanical Engineering.

† Research Assistant; now Assistant Professor, Southern Methodist University.

‡ Professor of Aerospace and Mechanical Engineering.

delineate dominant regimes for engineering purposes. For accelerating a flow by an area change, Moretti and Kays defined an acceleration parameter

$$K \,\Delta\, \frac{\nu}{u_\infty^2}\frac{\mathrm{d}u_\infty}{\mathrm{d}x}.$$

They suggested that for $K \gtrsim 3 \times 10^{-6}$ the flow would remain turbulent, while for higher values it was likely to revert to laminar.

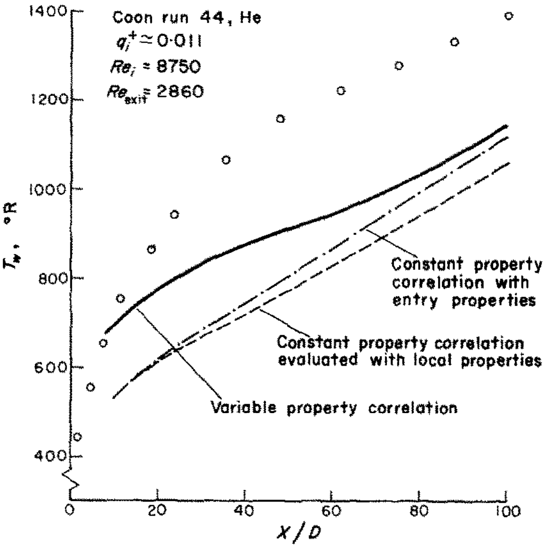

FIG. 1. Effect of premature relaminarization. Data of Coon [3] compared to predictions derived from typical turbulent correlations.

When gas is heated in a tube, the bulk Reynolds number drops axially and the viscosity increases near the wall. McEligot examined such flows and estimated their flow regimes from agreement with his turbulent and transitional correlations. The transitional runs were "strongly" heated ones in which the unheated entry flow was clearly in the turbulent range but which gave downstream results diverging from the turbulent correlations. He presented the estimated regimes as graphical functions of the parameters

$$q^+ \,\Delta\, \frac{q_w''}{GH_{i,0}} \approx \frac{q_w''}{Gc_pT_{i,0}} \text{ and } Re_i = \frac{4\dot{m}}{\pi D\mu_i}.$$

On Fig. 2 the approximate boundaries of the regimes are indicated by dashed lines. From data obtained by Perkins and Worsoe-Schmidt [7], an alternate presentation of the regimes was developed by McEligot *et al.* [8].

Bankston *et al.* [9] later presented additional data corroborating the work of [8]; still another classification scheme was included. In discussion, the question was raised whether the transition was due to acceleration; but it was concluded that the increase in viscosity was probably the cause. However, Bankston later found that some of their transitory runs had values of $K$ of the same order of magnitude as those of Moretti and Kays [4, 10]. In a recent thesis, Coon [3] has obtained additional data for retransition of a gas in heated tubes. He shows that his data are consistent with the flow regime classification of McEligot *et al.* and of Bankston *et al.* Further, he develops the acceleration parameter as follows: The bulk velocity is substituted for $u_\infty$ to give

$$K = \frac{\nu}{u_b^2}\frac{\mathrm{d}u_b}{\mathrm{d}x}.$$

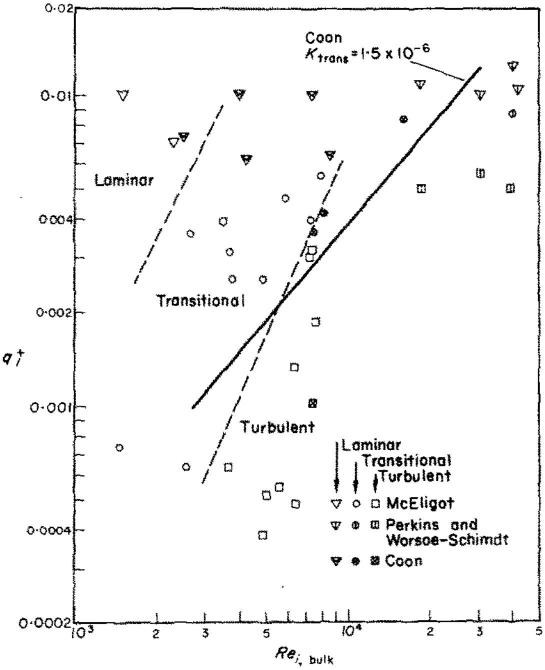

FIG. 2. Flow regime classification as estimated from correlation for transitional flow, □ = turbulent, ○ = transitional, ∇ = laminar, ▽ = doubtful (Extension of Figure VI-8 of [6]). Coon reverse transition criterion shown for comparison [3].

If the Mach number is low and the heating rate is high so that $(\mathrm{d}p/p) \ll (\mathrm{d}T/T)$, application of the continuity equation for a constant cross section and the perfect gas approximation leads to

$$K = \frac{\nu}{u_b}\frac{p}{\rho RT^2}\frac{\mathrm{d}T}{\mathrm{d}x}.$$

From an energy balance, noting that the kinetic energy terms are negligible (Coon's data were for Mach < 0·15), one solves for $\mathrm{d}T/\mathrm{d}x$ to obtain

$$K = \frac{4}{G^2 D} \frac{\mu}{T_b} \frac{q''_w}{c_p}.$$

The cycle is complete. When the definitions are compared, the acceleration parameter for the tube flow is seen to be

$$K = \frac{4q''_w}{Gc_{p,i}T_i} \frac{c_{p,i}}{c_p} \frac{T_i}{T} \frac{\mu}{\mu_i} \frac{\mu_i}{GD}$$

or

$$K = \frac{4q^+}{Re_i} \frac{\bar{\mu}}{\bar{c}_p \bar{T}_b}$$

or simply $4q^+/Re_i$ if evaluated at the start of heating. The three definitions of $K$ are seen to be equivalent when the bulk velocity is used in the Moretti and Kays parameter in place of the free stream external velocity and the assumptions noted above are made.* Thus, the parameters suggested by McEligot [6] form the acceleration parameter of Moretti and Kays [4]. Further, the approximate magnitudes suggested by McEligot are confirmed by Coon [3] (see Fig. 2). Other recent work, also plotted on Fig. 2, tends to show that the estimates of the flow regimes could be modified at high heating rates. (However, since we expect laminar flow to exist for $Re \gtrsim 2000$ under adiabatic conditions or low heating rates, the approximate regime boundaries should approach a lower vertical asymptote of $Re \approx 2000$.)

In the type of heated tube experiment discussed above, the wall heat flux is approximately constant for the entire heated length. Entering conditions and tube length determine whether $K_{\mathrm{trans}}$ is likely to be approached (recall that $Re_b$ drops as $x$ increases). For design estimates, the following relations, derived from an energy balance and definitions, apply for fluids with property variation expressible in power law form:

Constant wall heat flux (entering properties)

$$K \cong \frac{4q_i^+}{Re_i} \left[1 + 4q_i^+ \frac{x}{D}\right]^{a-d-1}.$$

Variable wall heat flux (local bulk properties)

$$K \cong \frac{8Q^+}{Re^2 Pr}.$$

The property-variation-exponent, $a$–$d$–1, is approximately $-0·43$ for air and $-0·35$ for helium at room temperatures and above. Hence, the first relation shows that only for very high heating rates or very long tubes does $K$ change substantially along a tube.

*In summary*, it is found that transition from turbulent to laminar flow can be *approximately* predicted for both internal and external flow experiments by use of the same parameter, $K_{\mathrm{trans}}$. And the order of magnitude is the same in both cases. However, underlying physical reasoning which would lead to this experimental observation is not yet resolved.

## ACKNOWLEDGEMENTS

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* Additional relaminarization criteria for adiabatic flow are presented in a recent paper by Patel and Head [11].